# Portable Pediatric Parallel Bars

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*Final Report*

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## *Executive Summary*

The primary objective of this project is to design and build a set of affordable, adjustable, and portable pediatric parallel bars for child patients aged 2-10 years. These portable parallel bars will allow travelling physical therapists (PTs) to transport their own parallel bars to the patient's home. This project is set in motion by our main client, Mrs. Shannon Smith; Mrs. Smith is a PT in Gainesville, who often works with outpatients in their homes. Mrs. Smith represents the travelling or site-based PT, who currently has no existing product to address the need of portable parallel bar therapy in the home. Our team regularly interacted with Mrs. Smith in order to update different project criteria and specifications according to the client's desires and requirements.

The Gaitway system provides the in-home pediatric physical therapist with a way to transport their parallel bars to their patient without losing the benefits of a fixed bar system. Through an innovative folding system and superior materials, the Gaitway can now service children from ages 2 to 10. The folding system requires 15 minutes at most for a full setup. The time spent with the patient is critical for rehabilitation, so the time saved in setup allows the physical therapist more time with their patient, which leads to a faster recovery. The Gaitway costs $350. This price actually saves the therapist money over time because they now have bars to accommodate 2-10 year olds; they no longer have to purchase redundant sets to service all their patients.

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## *Introduction*

The objective of this project is to design and build a set of affordable, adjustable, and portable pediatric parallel bars for children aged 2-10 years. These portable parallel bars will allow travelling physical therapists (PTs) to transport their own bars to the patient's home.

Parallel bars are used in the clinic to provide patients with a sturdy, consistent therapy tool that allows for repeatable exercise. Portable parallel bars are needed to allow outpatients this same comfort in sturdiness and consistency in their homes. Conversations with current PTs revealed that only two solutions exist to address the need for portable parallel bar therapy. The first solution is to build a set of bars using materials readily available to either the therapists or the parents of the patient. The most common material is PVC pipe, which can be found at most hardware stores. The second solution is an existing set of parallel bars, in fact made of PVC and supported with a plywood base. Neither of these solutions accommodate for the weight capacity and rigor of use required by PTs – they can only support children up to two years of age, and their stability decreases rapidly with repetitive use.

The solution is to build the bars in such a way as to make them sturdier and more portable to better serve the physical therapist and the child. We augmented the bars themselves to include bars to provide more support and stability. The ends of the bars also tighten into the upright supports to give additional support and perceived stability. The whole set of parallel bars folds onto itself which gives the bars a stable base and prevents excessive rocking.

## *Design Process Overview*

### *Specifications*

Table 1. Project specifications

| **Specifications** | **Source** |
|---|---|
| Project Criteria<br>- Need for portable parallel bars for at home physical therapy | Client |
| Sustainability Criteria<br>- 8 years of life-cycle | |
| User Criteria<br>- Child from 2-10 years of age with weight ranging from 15-120 lbs<br>- Product will be used in normal home setting | Client |
| Ergonomics Criteria<br>- Height of bars adjustable between 15 in and 30 in from ground<br>- Width between 13 in and 17 in<br>- Overall weight of product does not exceed 50 lbs | National Center for Health Statistics [1] |
| Functional Criteria<br>- Parallel bars can hold up to 150 lbs of force<br>- When bars are stowed, they take up to 30% ± 10% of full assembled volume | Client |
| Material and Physical Criteria<br>- Need to have protections against environment damage when being transported | |
| Aesthetics/Emotive Criteria<br>- Non-clinical look; use various bright colors for fun appearance (No medical white or blue, no sterile white and stainless steel combinations, and no utilitarian shape)<br>- Fits user demographic of children | |
| Jurisdictional Criteria<br>- Classified as “Sec. 890.5370 Non-measuring exercise equipment” as a class on medical device | Legal standards |
| Cost Criteria<br>- Cost to manufacture does not exceed $300 with profit margin of 20% | Market research |

As requested by our main client, Mrs. Smith, this product targets the user group of children up to 10 years of age. Since most children start gait training at the age of 2, this product’s target user group will be the children from 2-10 years of age. Therefore in order to maximize the possible use period, the life-cycle of this product is set to 8 years of use. Because this product targets for the use period of 8 years and because of the rapid physical changes associated with this user group in this period, a continuous height adjustability of this product is required.

Anthropometric research set a weight range of 15-120 lbs and a hip height range of 15-30 in, for ages of 2-10. The hip height is needed to determine the height of the parallel bars. This leads to the specification that the parallel bars need to have a continuous height adjustability from 15-30 in. Since this product will be mainly used in normal home settings, the length of the parallel bars will be no longer than 50 in to accommodate for the home setting. Based on anthropometric data, the width of the bars (horizontal distance between bars) will be set to 17 in. The product when stowed will be no larger or heavier than a checked baggage, which is 50 lbs and 62 in when the length, width, and height are added. This is because a checked baggage is a good weight and volume representation of what an average adult can transport. The design specification for the weight capacity will be 150 lbs, which means that the bars will be able to safely sustain a child of up to 150 lbs. In order to have an efficient stowage property for this product, the bars will be made so that they take up to 30% ± 10% of the full-assembled volume. The total cost of manufacturing of this product will be \$300 ± \$100 with a profit margin of 20% or up.

The appropriate aesthetics of the product for the targeted user group were also considered. The first aesthetical specification was that the parallel bars do not look medical. This means that the bars do not have any medical white or blue colors, and sterile white and stainless steel combinations for the material choice. The bars also should not be in a utilitarian shape. Considering the user demographic of children, a "toy" color palette, which includes various bright colors, will be used because this would encourage children to feel more comfortable about the parallel bars and about the gait training itself.

Limitations, standards and references from legal sources related to this product were investigated in order to consider the jurisdictional criteria of this project. Portable parallel bars are classified as a class one medical device, which means that they do not require a heavy premarketing application and regulation for FDA clearance to market. This product is considered as a class one medical device because it is intended to affect a function of the human body and it does not achieve its primary intended purpose through chemical action within the body (FDA). This project classifies as "Sec. 890.5370 Non-measuring exercise equipment" as a class one medical device (FDA).

During the design phase of this project, many designs with subsystems of different features were developed in order find the best way to accommodate with various criteria of the product. Various designs were considered and compared with sketches and prototypes, and they were converged to 6 sets of design concepts. The drawings and prototypes of designs are shown in the Appendix under the title "Design Overview". An evaluation matrix was used to select the best design based on the factors that affect the product. The evaluation matrix, titled Table 3. Evaluation matrix, is attached in the Appendix under title "Design Decision Process". The factors of strength, cost, maintenance, ease of assembly, manufacturability, aesthetics, simplicity, durability, safety, and portability were considered when evaluating the final concepts of designs. Strength, safety, and portability had the largest weight because those three factors are the most important factors that compose our value proposition of the product. The first design, called "tucked rails", had the advantage of having uniform bars and non-transforming base, thus increasing the strength of the whole system. However as shown in the evaluation matrix, it had low scores on manufacturability and portability due to its shelled part on the bottom and non-transforming bars and base. The second design, named "telescoping supports", had a low strength due to its telescoping parts but a high safety score with its uniform stanchions. Design 3, called “folded triangle”, had high scores on its safety and portability. This design is relatively strong and safe because of its side supports that prevent the stanchions from bending. It also had a high score on its portability because of its foldability in half. Design 4, called “sliding bars” had folding stanchions, telescoping middle supports and base, and sliding bars. Due to its transforming - both folding and telescoping - parts, it had low scores on strength and safety. The last design, named “folding bars”, had a high score on its ease of assembly because of its folding bars but had a relatively low score on its strength also because of its folding bars. During the process of final design selection, we realized the need of combining the best features of different designs to make one final design which best accommodates all. Design “folding bars” was ranked 1 with its high scores on strength, maintenance, ease of assembly, and portability. The final design was then developed with the base concept of “folding bars” but it also utilized the features from other designs. The concept of having uniform bars of “tucked rails”, which increases the strength and the safety, the concept of folding the base of “folding triangle”, which increases the portability and ease of assembly, and the concept of adjusting the height by translating side bars of “V shape”, which increases the simplicity, were utilized for the final design.

These specifications are all met by the final model. The height range is exactly 15-30 in, with a bar length of 4 ft and width of 18 in. The weight of the final model is 35 pounds and the stowed model takes up 23% of its open volume. Additionally, the parallel bars were selected out of aluminum specifically to withstand the weight of a child of 150 lbs. The total cost of the model was $300, which falls within the set range of $300 +/- $100. The colors were chosen to evoke whimsy; they are intentionally not medical looking to follow our emotive criteria. We also chose to go with a stain over a paint to allow the wood grain to show through. This preserves the warmth and softness of wood while also giving the overall structure a modernist feel.

## ***Precedents***

A market analysis was conducted to study and compare different products in order to better understand the current issues and the improvements that are needed for the product users. Parameters including the price ($), length (ft), capacity (lbs), lowest adjustable height (in), weight (lbs), portability, and sturdiness for current products were investigated. ‘Capacity’ refers to the weight limit of the bars, while ‘weight’ refers to the weight of the bars themselves. It was concluded that the capacity, sturdiness, portability, and weight were the more important parameters and a competitive landscape analysis was done with those parameters as shown in Figure 1. The target specifications to obtain are high sturdiness, high capacity, portability, and lightweight. As it is shown in the figure, currently there is no product in the market that satisfies all of these specifications. Most bars that are stable have a high capacity, but are far too heavy to be transported, or are fixed; most bars that are portable are either unstable or unwieldy due to the size and setup time.

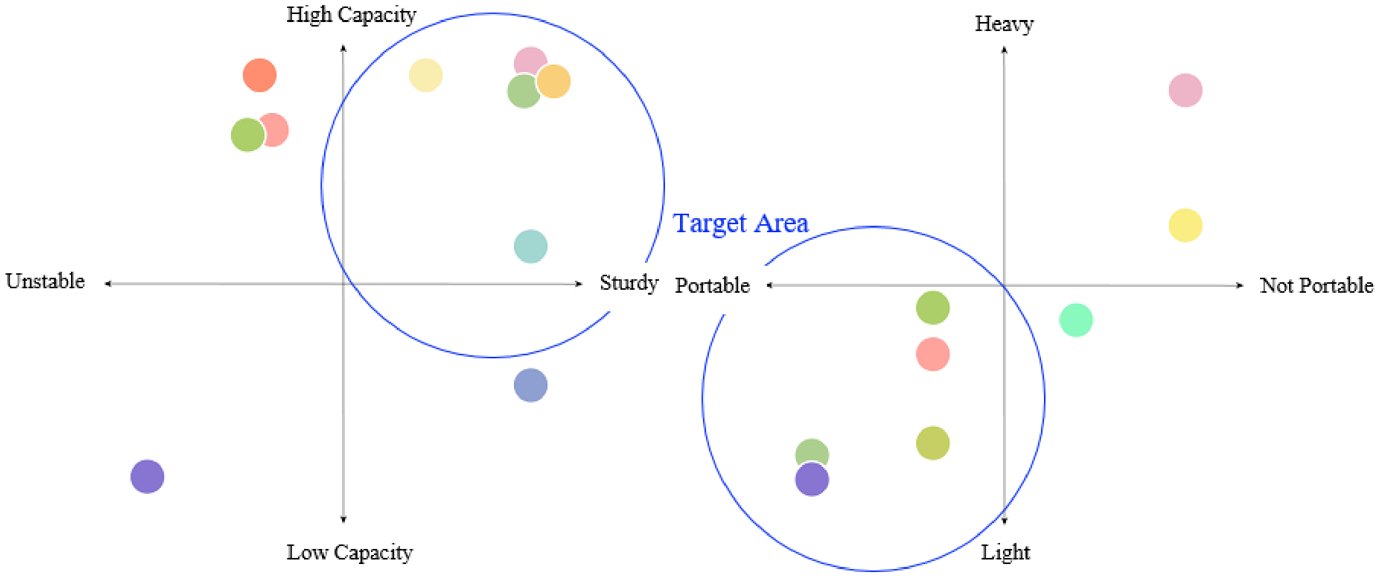


Figure 1. Competitive landscape market analysis

### *Value Proposition*

The Gaitway system provides the in-home pediatric physical therapist with a way to transport their parallel bars to their patient without losing the benefits of a fixed bar system. Current portable bars are not transportable, meaning they cannot be folded down to the size of a large suitcase which fits in most mid-sized sedans. Our Gaitway not only folds down to the size of a large suitcase, but does so while maintaining structural integrity.

Through an innovative folding system and superior materials, the Gaitway can now service children from 2 years to 10 years of age: that's a 70% increase over the leading competitor. The folding system requires fewer touchpoints to adjust, and 15 minutes at most for a full setup. The time spent with the patient is critical for rehabilitation, so the time saved in setup allows the physical therapist more time with their patient, which leads to a faster recovery. The Gaitway does cost $350, however this actually saves the therapist money over time because they now have bars to accommodate 2-10 year olds; they no longer have to purchase redundant sets to service all their patients.

### *Final Design Details*

We will introduce our final design of the Gaitway in this section. On this design, we exceeded all of our design requirements. The way that the supports are designed allows for the bar to be completely stable even under the maximum of 150 lbs of weight. A CAD modeling of the final design details and the detailed fabrication steps will also be introduced.

Figure 2 presents the whole system of the Gaitway. The uniform aluminum bars allow the users for a consistent interaction with the bars during physical therapy. Even though non-folding bars may decrease the ease of portability, they are still 4 feet which can be carried by one person and be fit into mid-sized sedans. Having uniform bars also allow for much higher strength of the whole system since this is where the force is directly applied by the user. Also, the stanchions (four red supports on each corner) are non-transforming and this critically increases the load that they can withstand before failing. T-tracks were used on stanchions for the bars to slide up and down during height adjustment and disassembly.

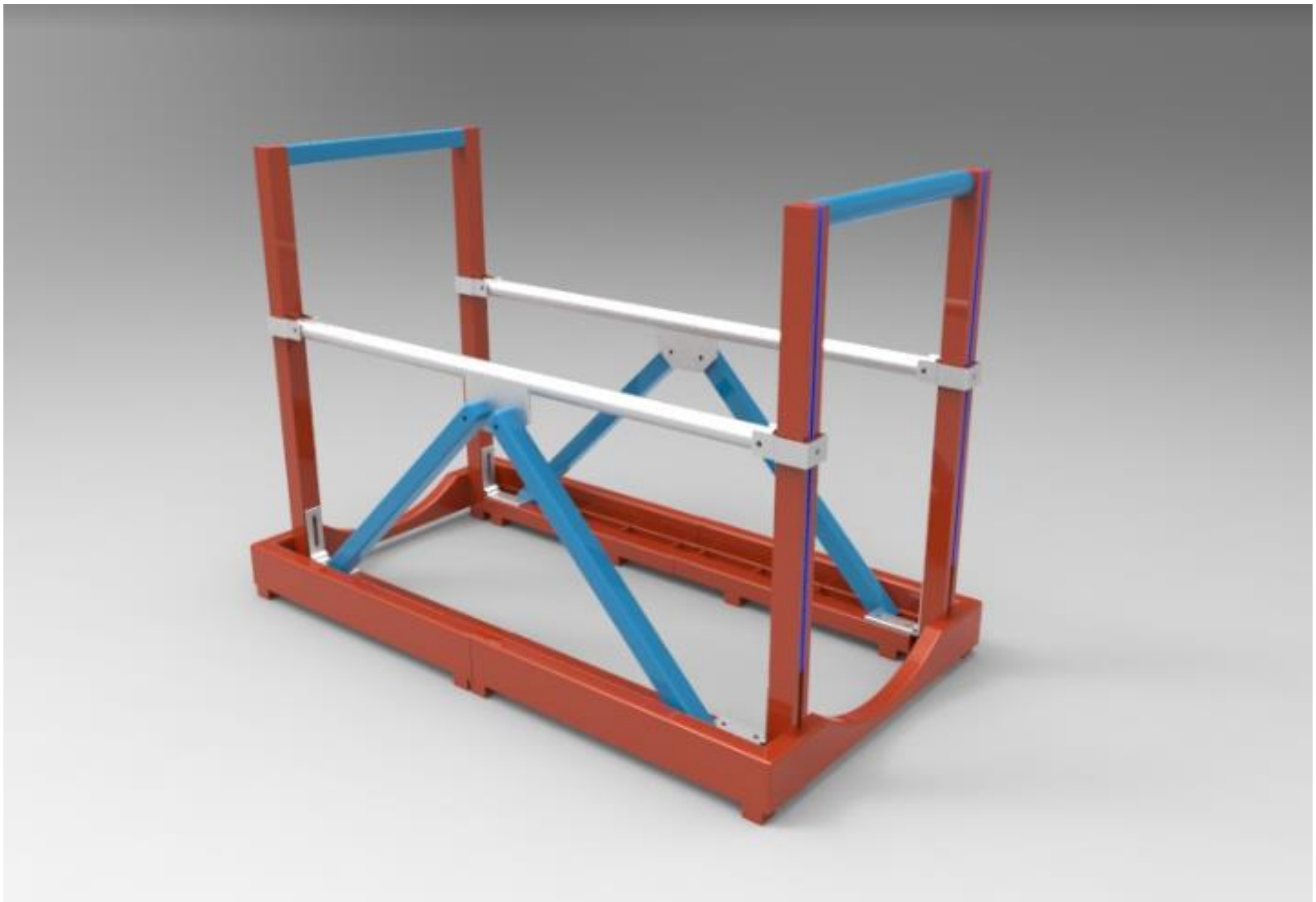


**Figure 2. Isometric view of the Gaitway**

Figures 3 and 4 show the height adjustment system of the Gaitway. The Gaitway utilizes the height adjustment mechanism of pool chairs. This mechanism allows the PTs to adjust the height in one motion and they do not have to adjust the height on each end of the bar. The assembly/disassembly process of the Gaitway is very simple. The horizontal bars are removed from the Gaitway by sliding them upward. The hinges between the stanchions and the base allow the stanchions to be folded downward. When they are folded, the base also folds in half, which makes the Gaitway in the size of a checked suitcase. This allows the PTs to easily disassemble and transport the Gaitway to different homes for their patients.

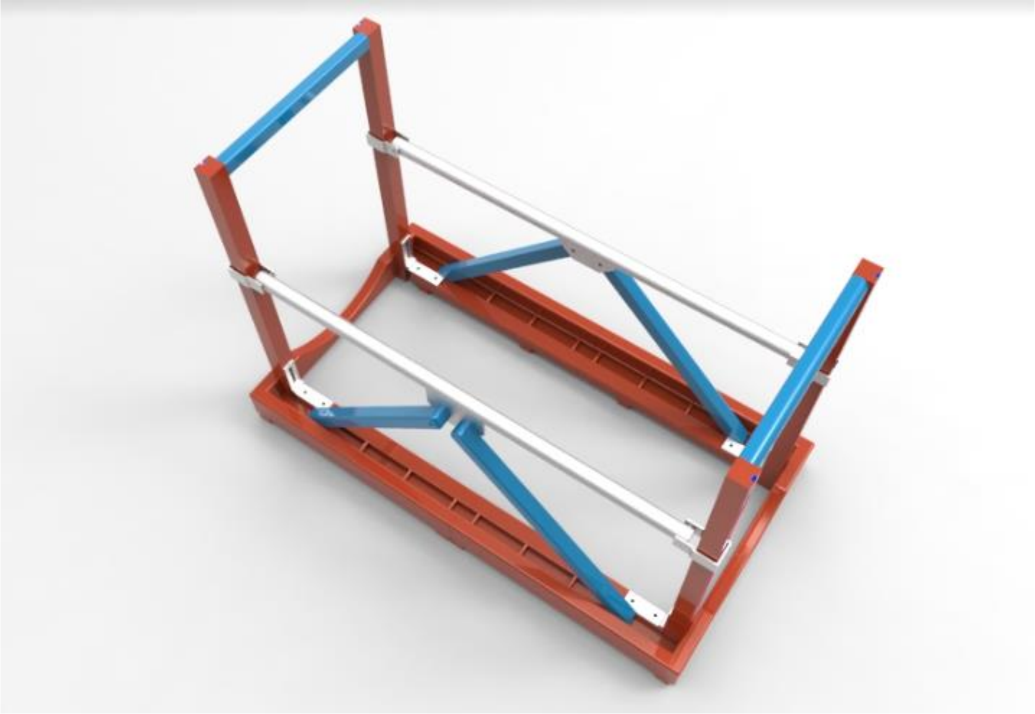


Figure 3. Top isometric view of the Gaitway

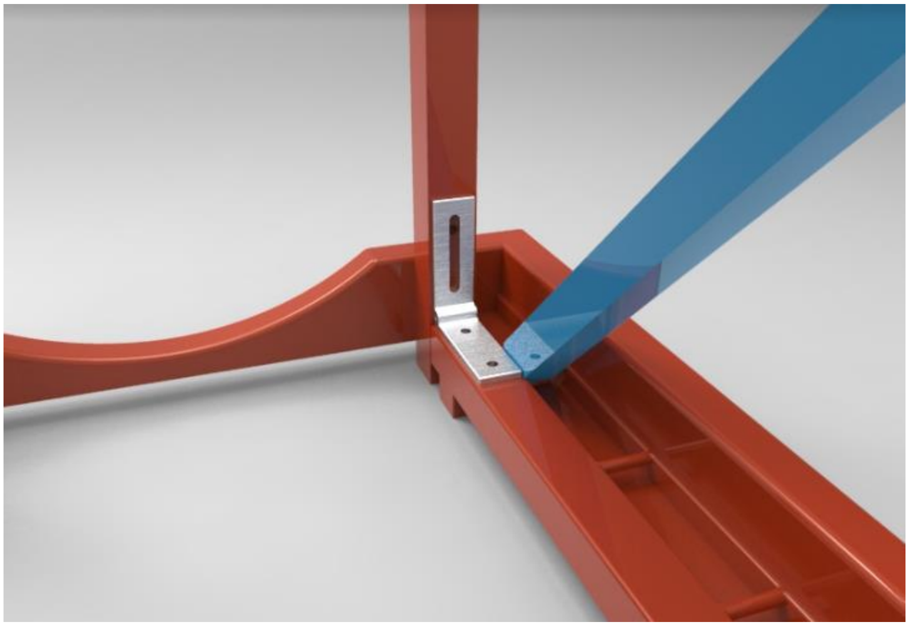


Figure 4. Height adjustment system of the Gaitway

## Fabrication:

Fabrication plan: To build this for full scale manufacturing it would need to be similar to how furniture is mass produced. The steps on how it would be manufactured is as follows:

1. Each half of the frame of the base is constructed. A semi-circle is cut out of the 25” piece to allow it to be more easily stepped over. At the same time, the four height adjustment ladders are being put together. Also happening at the same time is that aluminum tubes are being cut to 44” long. Aluminum sheets are being cut in a modified pentagon for the support bracket.
2. After the frame and the height adjustment ladders are assembled, a machine lines them up, glues, and pocket screws them together. One height adjustment ladder on each long side of the frame is added. The support bracket is welded onto the aluminum tube. At the same time, 1.75”x1.75” pine stock are being cut to 36” lengths, which form the stanchions. A channel is cut using a router out of the stanchions that hold up the parallel bar. This is the channel that the T-Tracks are installed into.
3. The T-Tracks are cut from 36” to 28” to account for the hinge.
4. The hinges are screwed onto the height adjustment ladders in such a place that they line up with the holes for the stanchions.
5. Hinges are installed on the bottoms of the frame of the base. The two parts of the base are held together with the hinge. There are 2 hinges, one on each side.
6. The T-Tracks are installed in the stanchions.
7. Sections of 3”x 10”x 0.125” sheet aluminum is cut for the brackets that attach to the end of the parallel bars. This sheet aluminum is then bent into shape and holes drilled to allow it to be attached to the rest of the assembly.
8. The stanchions are screwed onto the hinge using carriage bolts and lock nuts to allow the stanchions to slide up and into place.
9. T-Track screws are added onto the end brackets and the end brackets are screwed onto the ends of the aluminum parallel bar.
10. The Base (with the stanchions) are folded up and the stanchions are held against the side of the base using bungee cords.

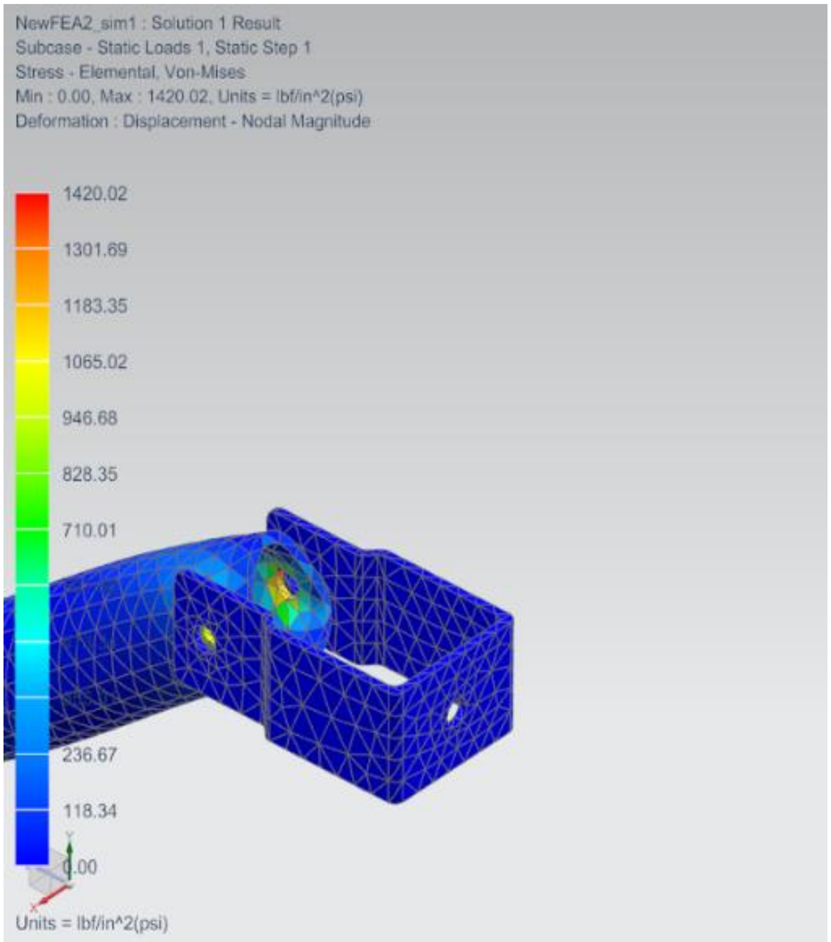
**Bill of Materials:**

| **Part Number** | **Amount** | **Part Name** | **Part Dimensions** | **Part Cost (per unit)** | **Part Manufacturer** |
|---|---|---|---|---|---|
| 1 | 4 | Pine Board | 2’x4’x8’ | 3.01 | Home Depot |
| 2 | 2 | Pine Board | 2’x2’x8’ | 1.96 | Home Depot |
| 3 | 6 | Steel Hinges | 1.5”x4”x0.25” | 13.81 | McMaster |
| 4 | 4 | Pine Dowels | 1.75”x1.75”x36” | 7.98 | Home Depot |
| 5 | 4 | Aluminum T-Tracks | 0.75”x0.25”x36” | 19.99 | Rockler |
| 6 | 4 | T-Track Bolts | 5/16”x2” | 1.49 | Rockler |
| 7 | 4 | Cross Handles | 2” Diameter | 2.49 | Rockler |
| 8 | 2 | Aluminum bars | 1.5”x84” | 47.66 | McMaster |
| 9 | 1 | Aluminum Sheet | 0.125”x4”x12” | 4.59 | McMaster |
| 10 | 1 | Steel Sheet with Holes | 2”x0.125”x48” | 6.51 | Home Depot |
| 11 | 16 | Lag Bolts | 9/16”x1.5” | 0.38 | Home Depot |
| 12 | 8 | Carriage Bolts | 9/16”x2” | 0.46 | Home Depot |
| 13 | 8 | Nylon Lock Nuts | 9/16” | 0.12 | Home Depot |

### *Design Performance*

A mathematical engineering analysis was performed in order to verify that the Gaitway meets and exceeds the desired mechanical specifications. By treating one side of the Gaitway as one horizontal bar simply supported by two identical vertical bars, the critical forces of the horizontal and the vertical bars were calculated. The critical force is the maximum force which a bar can withstand while staying straight. The process of calculation is shown in the Appendix under the title "Engineering Analysis of Simply Supported Parallel Bars". The critical force of the horizontal bar was calculated to be 772.4 lbf, which is much larger than our maximum weight criteria of 150 lbf. The safety factor, which is the load withstanding capacity of a system beyond the expected loads, was calculated to be 7. With the safety factor and the result of the critical force, it is concluded that the Gaitway is safe to be used under the expected load.

The final design of the Gaitway was tested and analyzed through the method of finite element analysis (FEA) by using software, NX 10.0. FEA was utilized in order to investigate the stresses acting on the joints between the bars and the stanchions during operation with the maximum force applied. The result of Von-Mises stress shown in Figure 5 was obtained. As shown in the contour plot, the maximum stress occurs at 1420 psi in the bolted joint between the bar and the joint during operation. This makes sense because the hole is where the stress concentration is. By using the general yield strength of 39,885 psi of steel, which was the material that was used for this joint, it was concluded that it is still safe to use this joint for the maximum weight that is expected to be applied to the Gaitway.



```
NewFEA2_sim1 : Solution 1 Result
Subcase - Static Loads 1, Static Step 1
Stress - Elemental, Von-Mises
Min : 0.00, Max : 1420.02, Units = lbf/in^2(psi)
Deformation : Displacement - Nodal Magnitude
```

Figure 5. Von-Mises stress on joints

A hand analysis was also done for the bolts that were used for the joints between the horizontal bars and the stanchions. An SAE Grade-1 5/16 bolt requires a preload of 1297 lbf (Budynas, Equation 8-31). This translates to a torque of 6.75 ft-lbf, for a galvanized steel bolt (Budynas, Equation 8-27). The bolt is in a slip-critical joint, meaning that its strength is dependent on the friction between the bolt and the t-slot. When a critical shear force exerted on the bolt overcomes this friction, the bolt will slip. A critical shear force is calculated as 290 lbf (RSCS, Equation 5.7) for uncoated faying surfaces and for a bolt tightened along long slotted holes parallel to the load direction. This gives a safety factor of 1.9 for the intended load of 150 lbf. It must be noted that this slip resistance is only valid as long as the bolt is correctly tightened to a preload of 1297 lbf.

### *Environment and Sustainability Considerations*

The design of the Gaitway was also developed with the consideration of the impacts that it will bring in the areas of environment and sustainability. The Gaitway is more environment friendly, when compared to the PVC parallel bars that most of the PTs previously used, with a much longer life-cycle of 8 years than the expected life-cycle of about 2 years of the PVC parallel bars. This extended life-cycle will prevent the buyers from purchasing the parallel bars more frequently thus preventing the manufacturers to use more materials more frequently as well.

Also, most of the parts that do not need to endure direct force are made with pine wood. This helps with raising the sustainability consideration of the Gaitway because pine wood is one of the great sustainable materials that you can find in the market. Sustainable wood, such as pine wood, comes from sustainably managed forests. Sustainable wood is renewable because the forest stewards manage the landscape to prevent damage to eco-systems, wildlife, watersheds, and the trees themselves. The use of pine wood saves both the material cost and the environment.

### *Realization and Deployment*

To fully realize this design, more thought must go into the joint where the uprights attach to the base. Currently as they are, they have a problem where if too much force is put on the uprights the supports can deform and cause the supports to be able to wiggle back and forth.

To deploy this product to the customers we would need to contract a manufacturer that would be able to make all the parts with very tight tolerances.

### *Conclusion*

There are many benefits and advantages that the Gaitway will bring to traveling physical therapists. It will provide both sturdiness and portability which none of the competing equipment in the current market has. When desired user specifications and the design performance of the Gaitway are considered, it can be concluded that it not only meets but exceeds the desired criteria defined by the user and the client.

The uniform bars and supports of the Gaitway provide the strength and safety, the folding hinges and base provide the portability, the height adjustment mechanism provides the ease of adjustability, the sliding tracks on the stanchions provide the ease of assembly/disassembly, and the color selection provides the aesthetical strength. The uniform bars allow the user to have a consistent feel throughout the exercise and they also critically increase the maximum force that they can withstand before failing. The folding hinges and the folding base allow the Gaitway to be flat on the base and to be folded in half to be the size of a checked suitcase. This makes it a lot easier for PTs to transport the Gaitway to different homes of their patients. The red and blue color selection, which reminds of toy colors, allows the user to be more comfortable about the Gaitway and about gait training itself. With all of these innovative systems applied, the Gaitway now provides the in-home pediatric physical therapist with a way to transport their parallel bars to their patient without losing the benefits of a fixed bar system.

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## *Contacts*

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## *Appendix*

Anthropometric Research

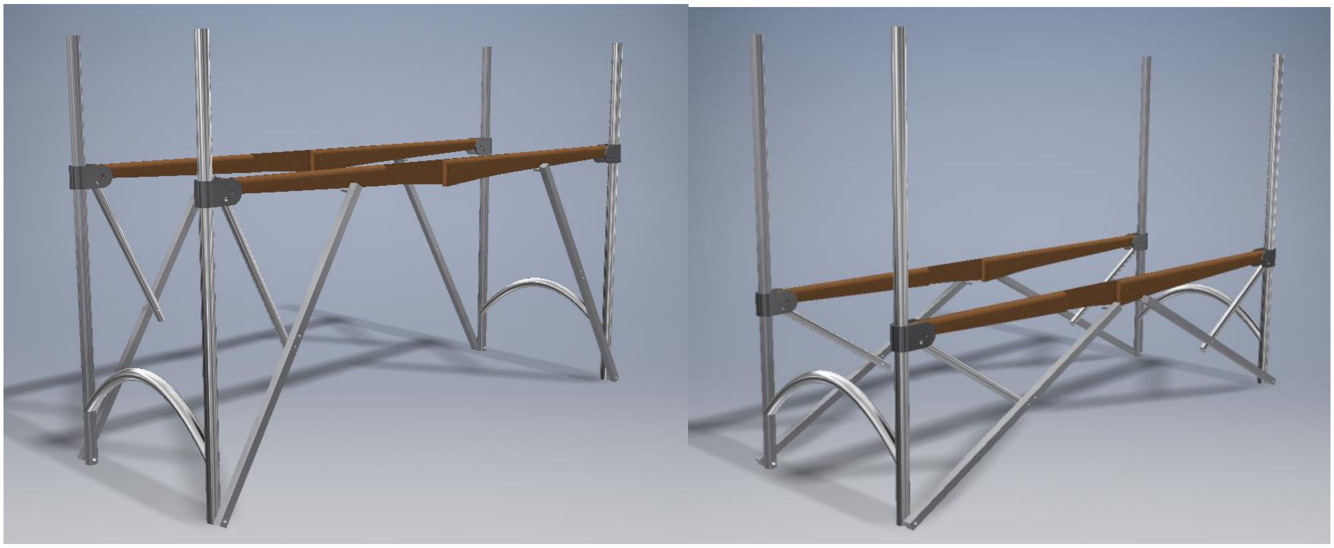
### Table 2. Anthropometric Data

| **Age** | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **Ratio**[1] | 1.7 | 1.6 | 1.6 | 1.5 | 1.4 | 1.4 | 1.3 | 1.2 | 1.1 | 1.1 | 1.0 |
| **Male Height (in)**[2] | - | - | 38 | 41 | 44 | 48 | 50 | 53 | 55 | 58 | 59 |
| **Female Height (in)** [2] | - | - | 38 | 41 | 43 | 47 | 50 | 52 | 55 | 57 | 60 |
| **Estimated Hip Height (in)** | - | - | 15 | - | - | - | - | - | - | - | 30 |
| **Weight (lb)** [2] | - | - | - | - | - | - | - | - | - | - | 121 |
| **Biacromial Breadth (in)** [3] | 8.7 | - | - | - | - | - | - | - | - | - | 12 |

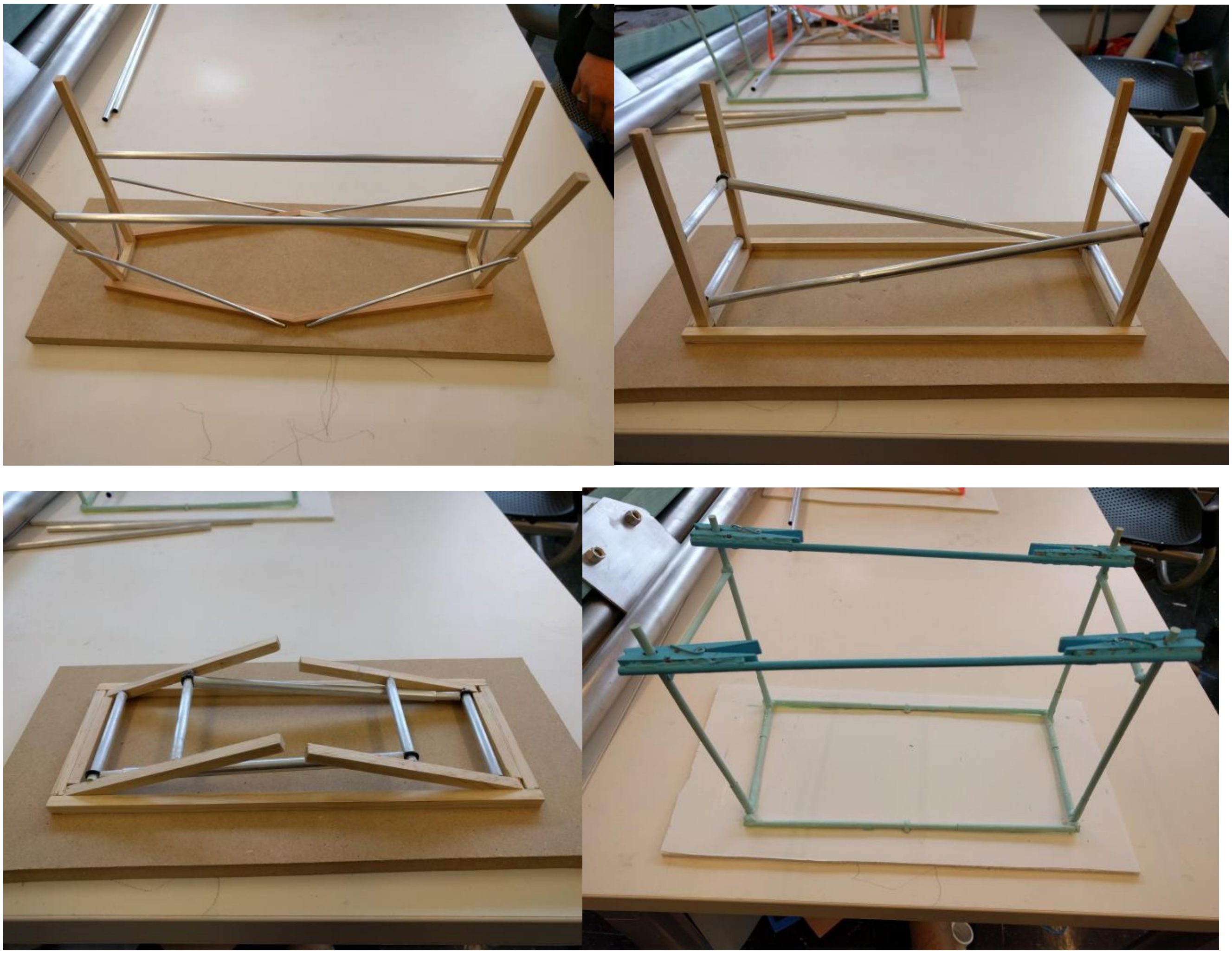
[1] (Nwosu) [2] (Fryar) [3] (McDowell)

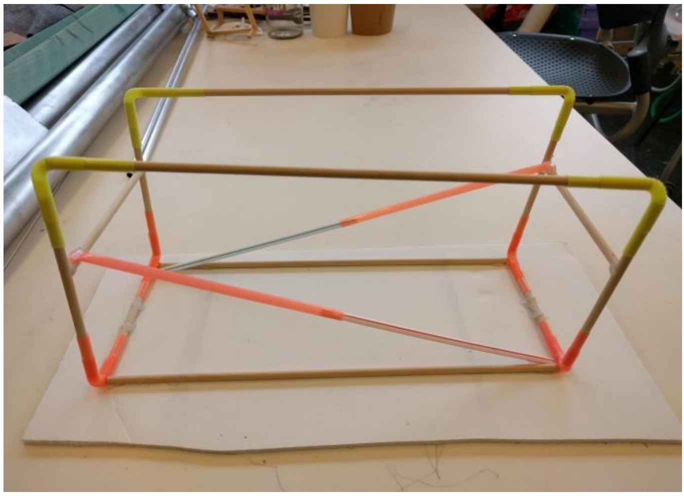
## Design Overview

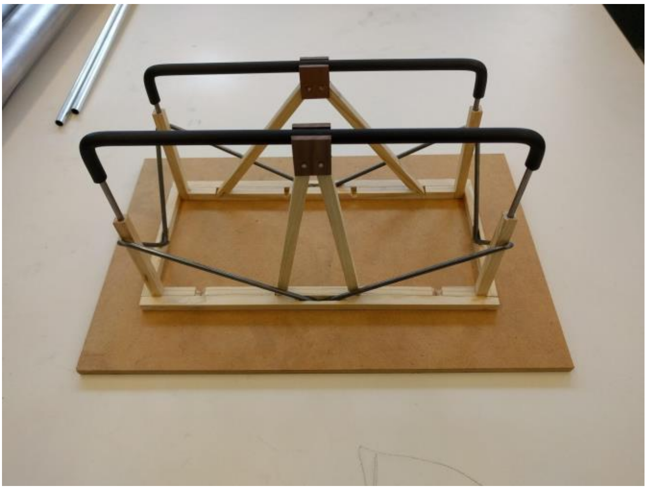
First CAD model prototype:

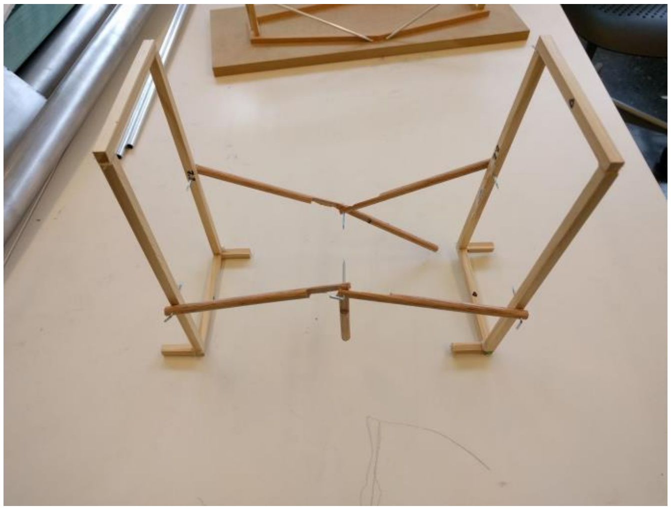


Prototypes:





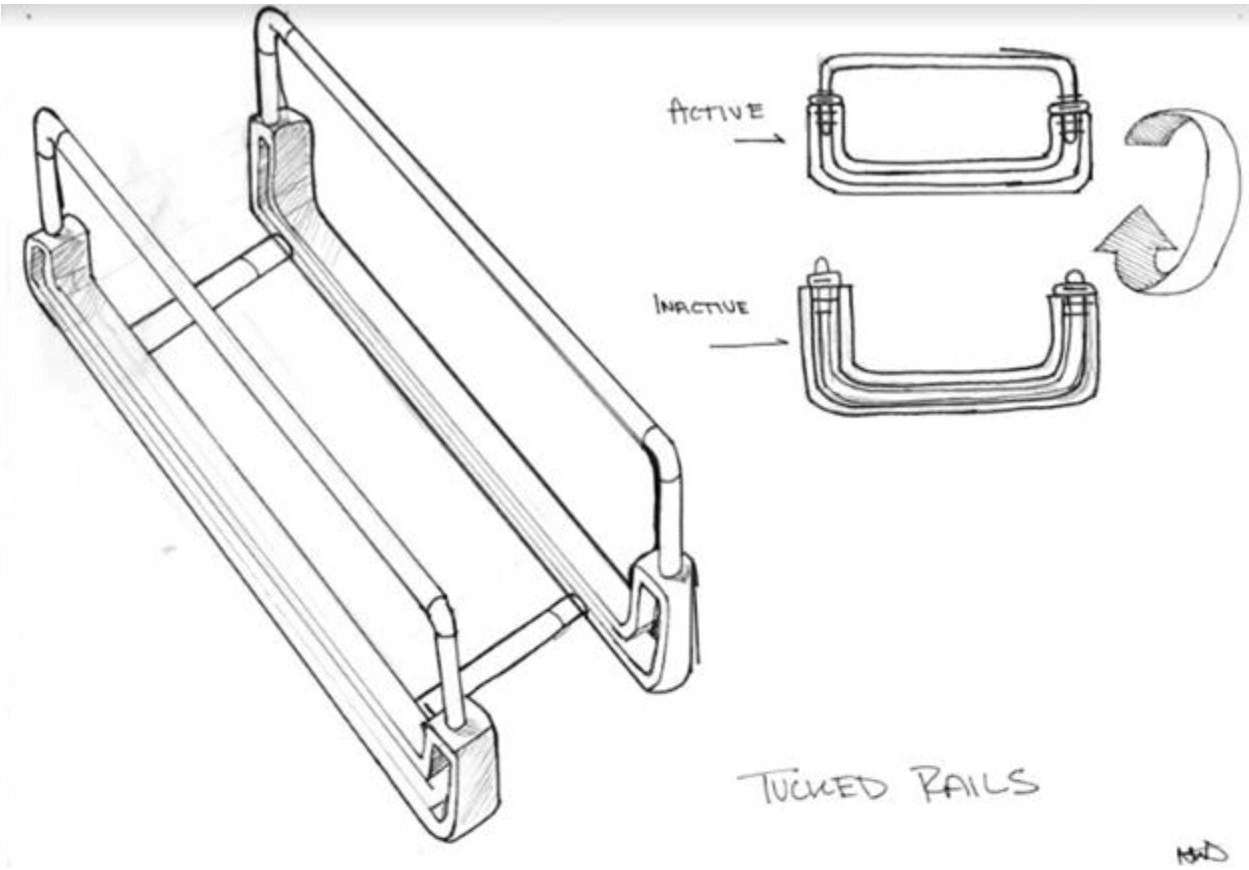




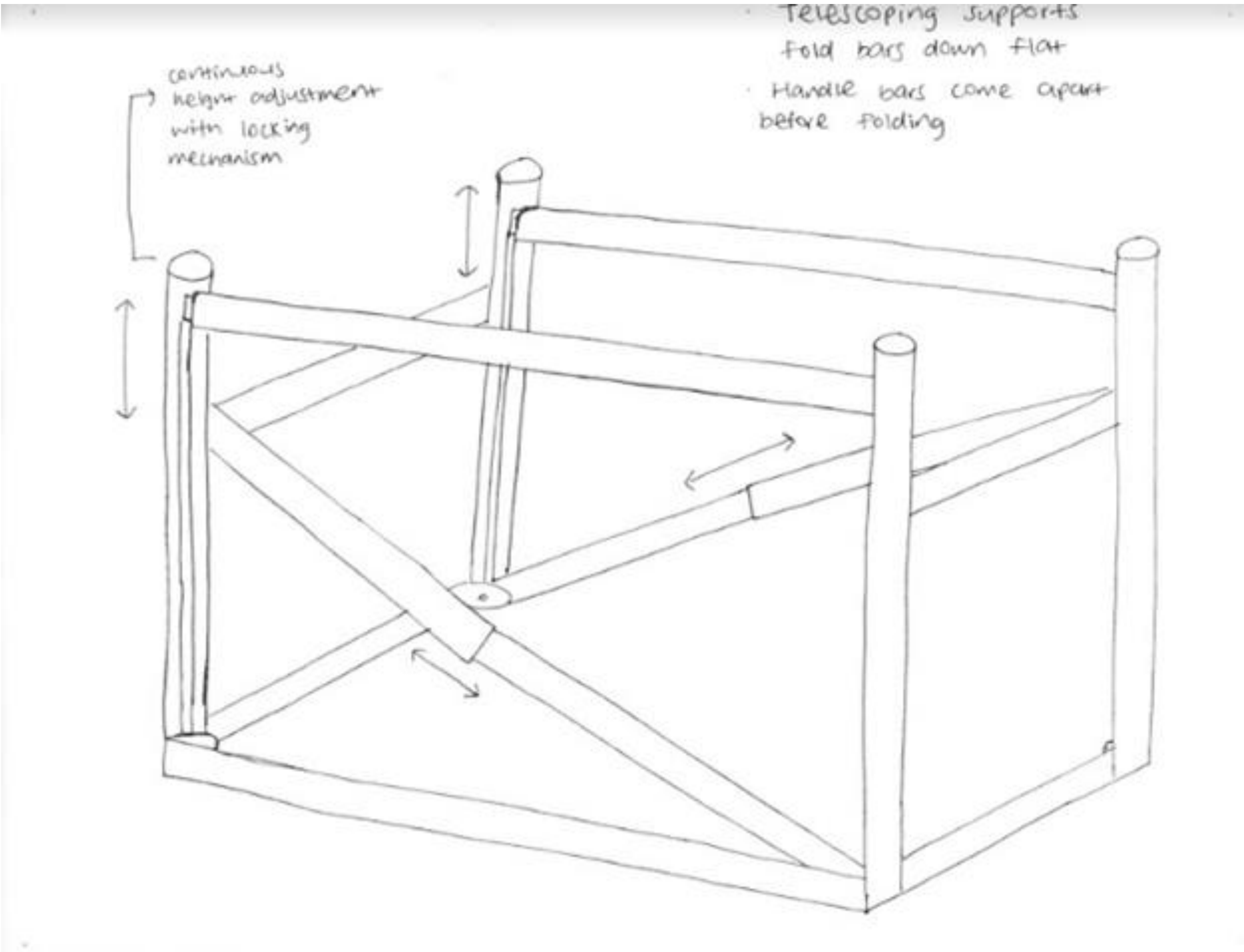
Design Decision Process

**Table 3. Evaluation matrix**

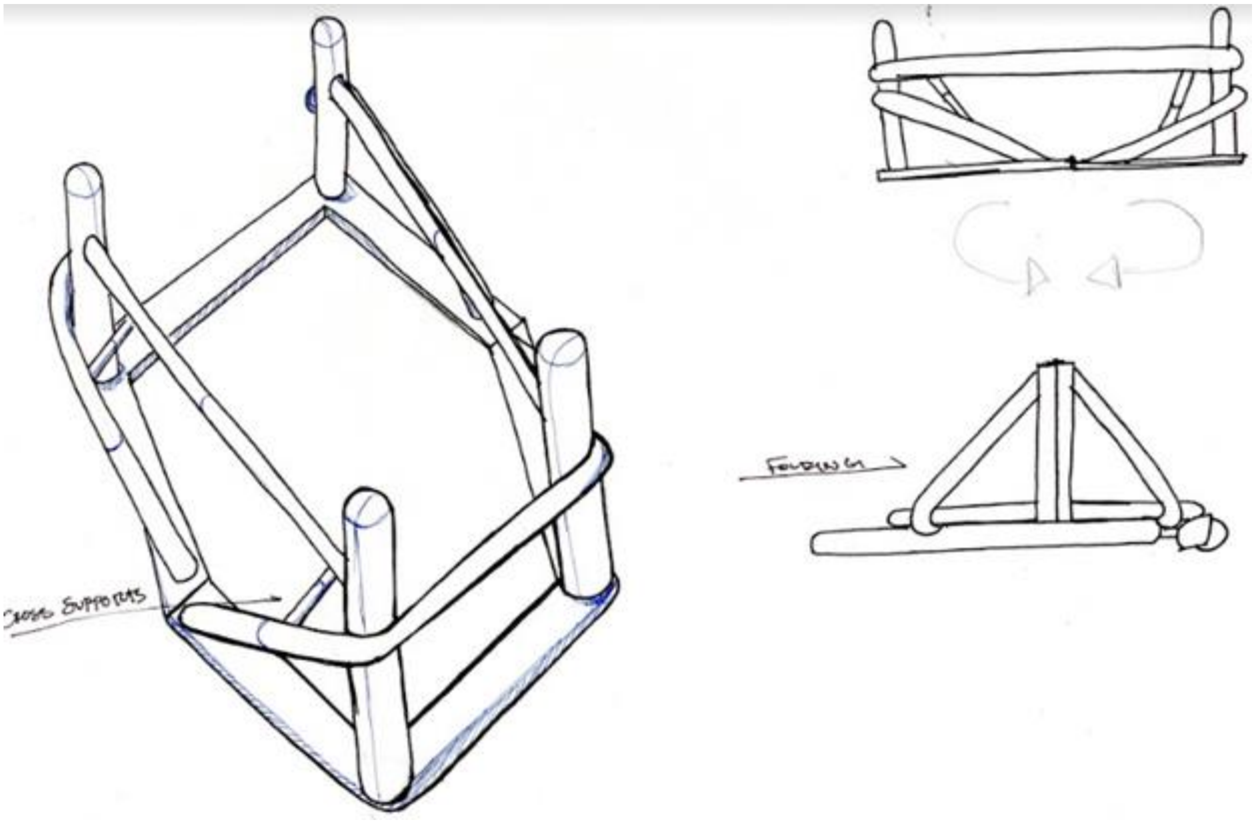
| | | **Tucked rails** | **Telescoping supports** | **Folded triangle** | **Sliding bars** | **V Shape** | **Folding bars** |
|---|---|---|---|---|---|---|---|
| **Factors** | **Weight** | | | | | | |
| **Strength** | 10 | 5 | 3 | 4 | 2 | 3 | 4 |
| **Cost** | 6 | 2 | 4 | 4 | 3 | 5 | 4 |
| **Maintenance** | 3 | 3 | 4 | 3 | 2 | 3 | 5 |
| **Ease of assembly** | 8 | 4 | 3 | 3 | 1 | 4 | 5 |
| **Manufacturability** | 6 | 2 | 4 | 4 | 3 | 5 | 4 |
| **Aesthetics** | 5 | 4 | 3 | 4 | 1 | 2 | 4 |
| **Simplicity** | 4 | 5 | 4 | 3 | 2 | 5 | 3 |
| **Durability** | 9 | 3 | 2 | 3 | 3 | 3 | 4 |
| **Safety** | 10 | 4 | 5 | 5 | 2 | 3 | 3 |
| **Portability** | 10 | 1 | 1 | 5 | 2 | 4 | 4 |
| | **Total Score** | 232 | 223 | 240 | 185 | 258 | 281 |
| | **Rank** | 4 | 5 | 3 | 6 | 2 | **1** |
| Weights rated 1-10<br>Rating based on 1 (worst) – 5 (best) | | | | | | | |



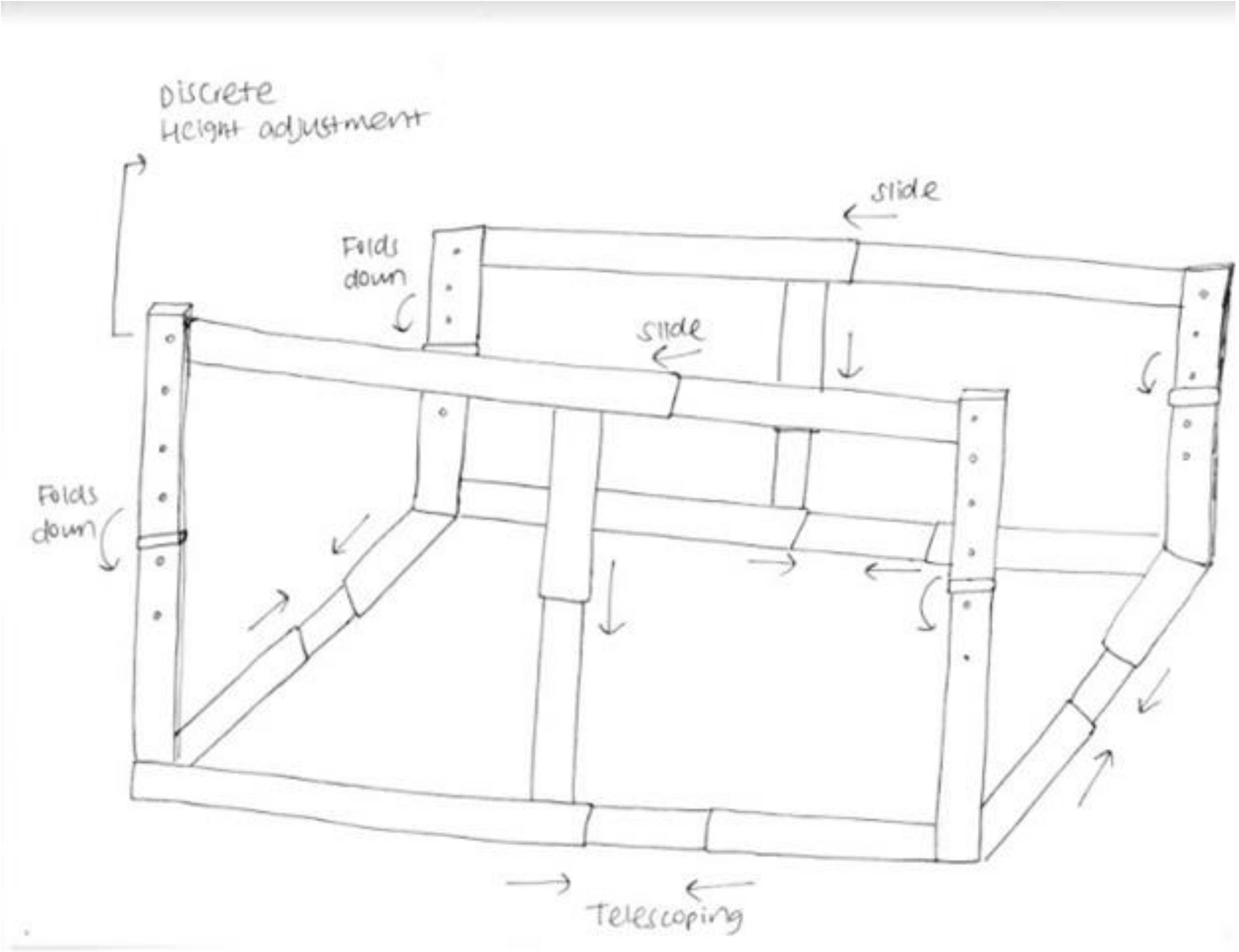
**Design 1. Tucked rails**



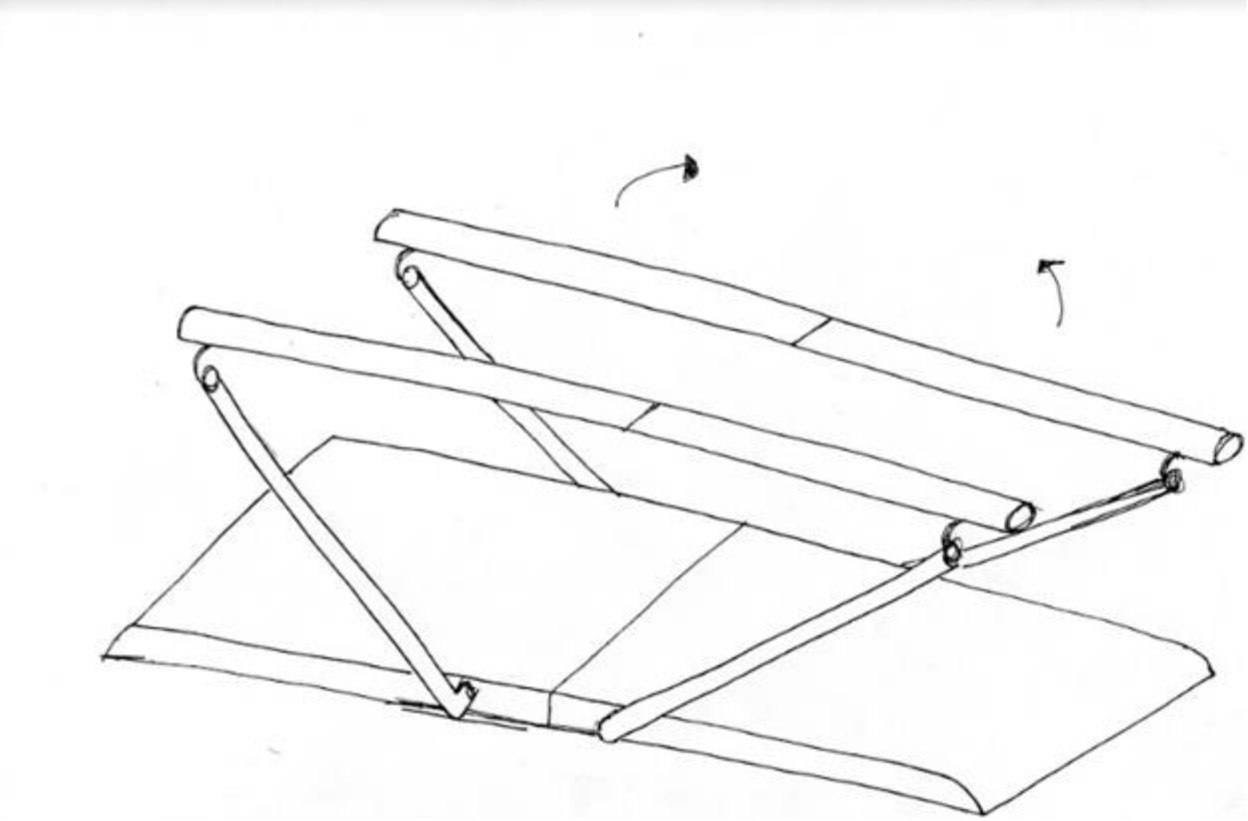
**Design 2. Telescoping supports**



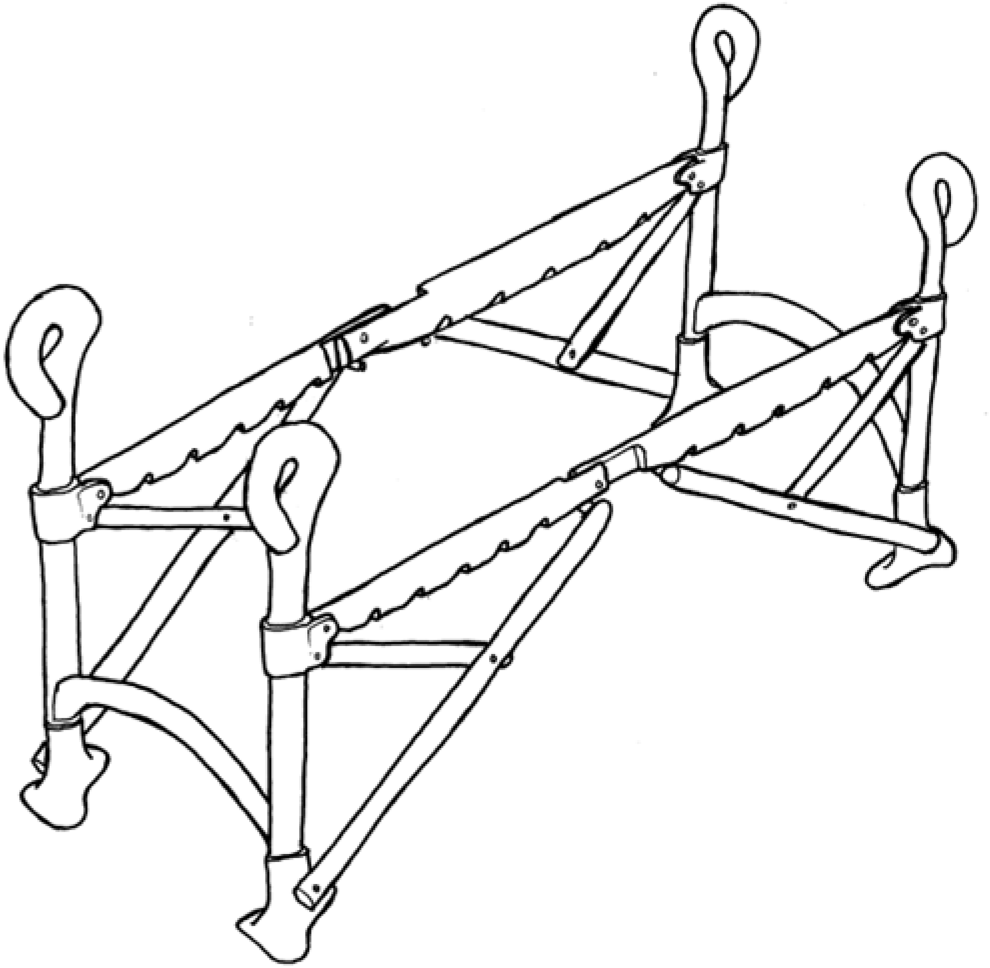
**Design 3. Folded triangle**



Design 4. Sliding bars

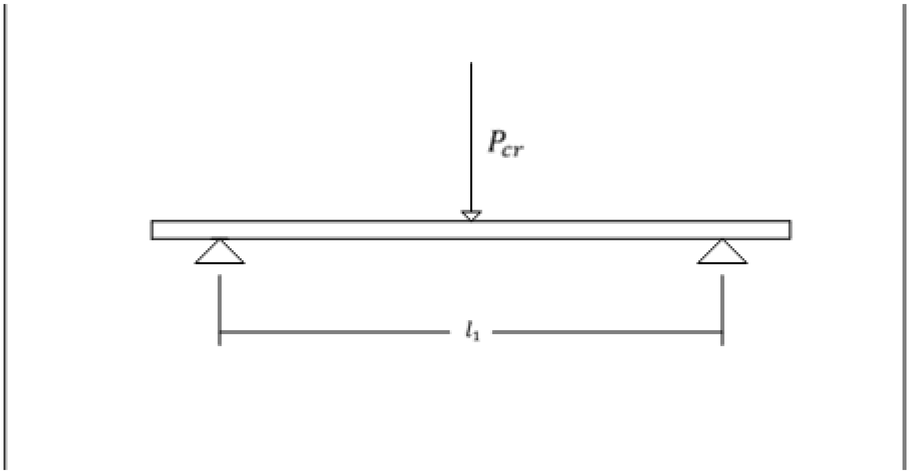


**Design 5. V Shape**



Design 6. Folding bars

Engineering Analysis of Simply Supported Parallel Bar



The maximum bending moment $M_{max}$induced by $P_{cr}$is given by Equation 1. It occurs at the midpoint of $l_1$, equidistant from both supports.

$$M_{max} = \frac{P_{cr} l_1}{4} \tag{1}$$

At the same point, the shear force $V$ is given by Equation 2.

$$V = \frac{P_{cr}}{2} \tag{2}$$

From the maximum bending moment $M_{max}$ we can find the maximum bending stress $\sigma_{max}$, given by Equation 3.

$$\sigma_{max} = \frac{M_{max} c}{I} \ where\ c, I\ depend\ on\ A_c \tag{3}$$

By substitution:

$$\sigma_{max} = \frac{P_{cr} l_1 c}{4I} \tag{4}$$

The transverse shear force at any section cut can be found using Equation 5.

$$\tau_r = \frac{VQ}{Ib} \tag{5}$$

By substitution:

$$\tau_r = \frac{P_{cr} Q}{2Ib} \ where\ Q, b, I\ depend\ on\ A_c \tag{6}$$

The bending stress and transverse shear are however *not* maximum stresses present in the bar. We must solve for principal stresses. The general principal shear stress is given by Equation 6. Note: the principal shear stress depends on the *maximum* value of transverse shear.

$$\tau' = \sqrt{(\frac{\sigma_{max}}{2})^2 + {\tau_{r,max}}^2} \tag{7}$$

By substitution:

$$\sqrt{(\frac{P_{cr} l_1 c}{8I})^2 + (\frac{P_{cr} Q}{2Ib}|_{max})^2} \tag{8}$$

The general principal normal stress is given by Equation 7.

$$\sigma' = \frac{\sigma_{max}}{2} + \tau' \tag{9}$$

By substitution:

$$\sigma' = \frac{P_{cr} l_1 c}{8I} + \sqrt{(\frac{P_{cr} l_1 c}{8I})^2 + (\frac{P_{cr} Q}{2Ib}|_{max})^2} \tag{10}$$

From these principal stresses, we can determine $P_{cr}$ using appropriate failure theories.

We select conservative failure theories. There can be three possible theories, dependent on material selection.

1. If $\varepsilon_f < 0.05$ then we proceed with Brittle Coulomb-Mohr theory.
2. If $\varepsilon_f > 0.05$ and $S_{ut} \neq S_{yc}$ then we proceed with Ductile Coulomb-Mohr theory.
3. If $\varepsilon_f > 0.05$ and $S_{ut} = S_{yc}$ then we proceed with Maximum Shear Stress (MSS) theory.

Most likely, we will face the third situation.

For MSS,

$$\tau' = \frac{S_y}{2n} \tag{11}$$

where $n$ is a safety factor chosen by the engineer. We select $n = 2$.

By substitution:

$$\sqrt{\left(\frac{P_{cr} l_1 c}{8I}\right)^2 + \left(\frac{P_{cr} Q}{2Ib}\Big|_{max}\right)^2} = \frac{S_y}{4} \qquad (12)$$

Rearranging terms:

$$P_{cr} = \frac{S_y}{4\sqrt{\left(\frac{l_1 c}{8I}\right)^2 + \left(\frac{Q}{2Ib}\Big|_{max}\right)^2}} \qquad (13)$$

By using Equation 13 and the geometry and the material properties of our horizontal bars, we can solve for the critical force that the horizontal bars of Gaitway can withstand.

By substituting the geometrical and material properties shown in Table 4 below, the critical force is calculated to be 772.4 lbf.

**Table 4. Geometry and material properties of Aluminum 6061 bar for Gaitway**

| | |
|---|---|
| **Yield strength of Aluminum 6061, $S_y$** | 35 kpsi |
| **Length of horizontal bar, $l_1$** | 4 ft |
| **Centroid of cross-sectional area, c** | 0.75 in |
| **Second moment of area, *I*** | 0.20 |
| **Top portion of cross-sectional area, *A* * distance to the centroid of *A*** | 0.982 |
| **Sectional width, *b*** | 1.96 |
| **Critical force, $P_{cr}$** | 772.4 lbf |